

Geomechanics

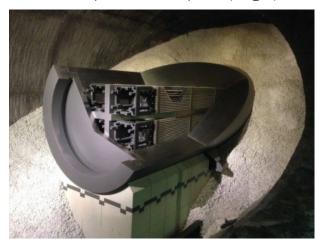
LECTURE 11

GEOMECHANICAL BEHAVIOUR OF UNSATURATED GEOMATERIALS

PROF. LYESSE LALOUI

Laboratory of soil mechanics - Fall 2024 25.11.2024

Swiss concept for HLW disposal (Nagra)

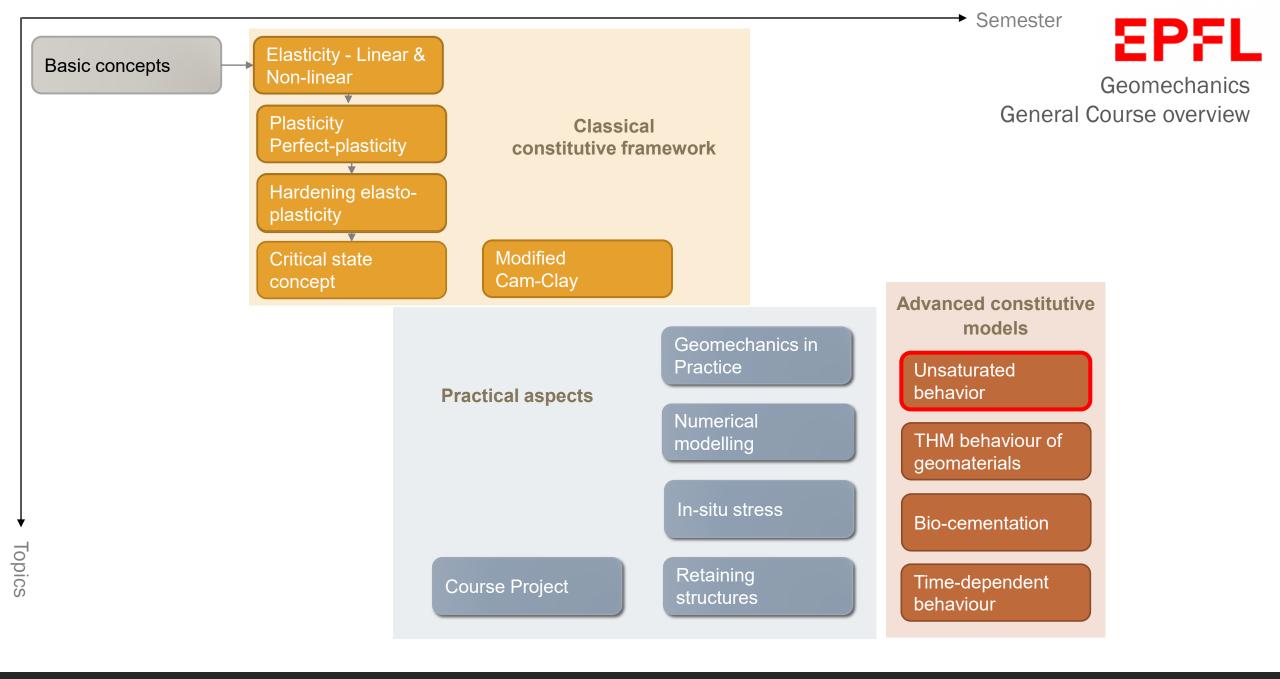




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Content



- Basic concepts
- Hydraulic properties of geomaterials under unsaturated conditions
- Mechanical behaviour of unsaturated geomaterials
- Conclusion



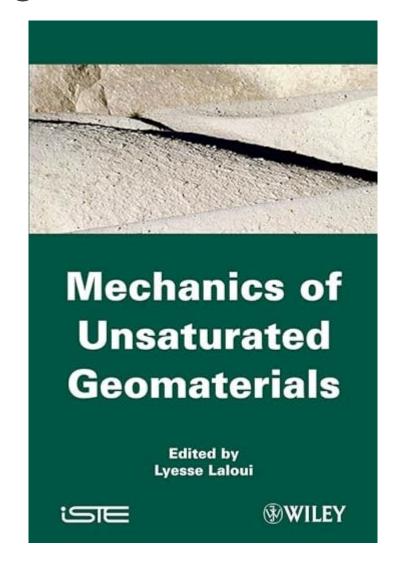
Basic concepts

INTRODUCTION

THE DEFINITION OF SUCTION

Useful reference



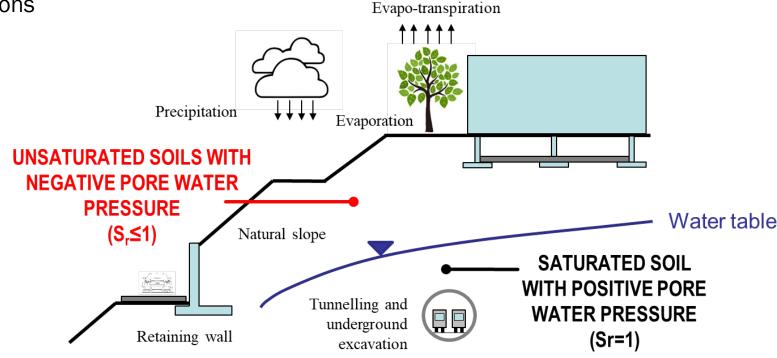




Saturated and unsaturated states in practice

- Embankments and tunnels
- Road foundations
- At ground surface (infiltration and evaporation)
- Geoenvironmental engineering applications (nuclear and domestic waste storage)

Hydro-mechanical properties of the soil need to be characterized in saturated and in unsaturated conditions



Above the water table the pore water pressure is negative, and the soil can become unsaturated



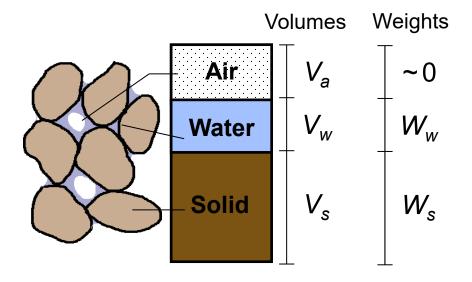
Unsaturated soils

3 phases

- Gas phase (air)
- Liquid phase (water)
- Solid phase (solid particles)

3 stress variables

- σ total mechanical stress
- p_w porewater pressure
- p_a pore-air pressure



Three-phase description

Many ways to quantify the amount of water

- Gravimetric water content
- Degree of saturation
- Volumetric water content
- Water ratio

$$W = W_w/W_s$$

$$S_r = V_w/Vv$$

$$q = V_w/V$$

$$e_w = V_w/Vs$$

Volume of voids
$$V_v = V_a + V_w$$

Total volume
$$V = Vv_{+} Vs$$

Porosity
$$n = Vv/V$$

Void ratio
$$e = Vv/Vs$$

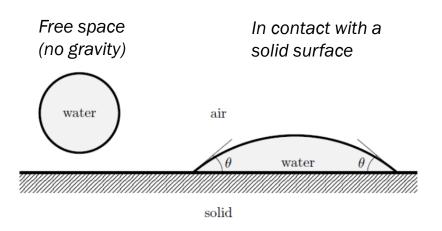


Complex interaction between water, air and solid phases

Surface tension, T:

- energy per unit area of the surface between phases
- energy penalty of breaking the intermolecular interactions between the two phases, and instead creating an interface between them

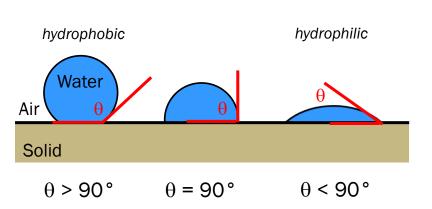
Fluid arrangements



Representative values Air/water: 72 mN/m Oil/water: 50 mN/m Air/oil: 20 mN/m

Contact angle, θ :

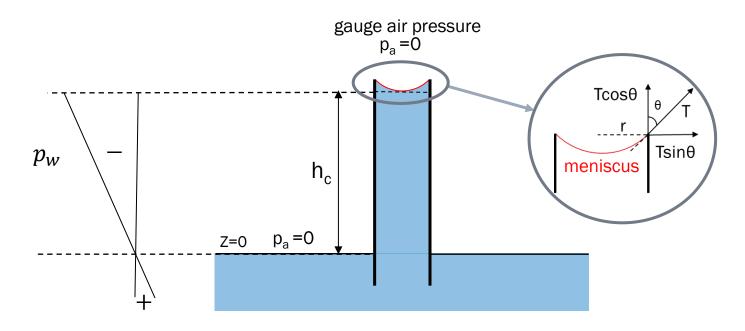
- angle between two fluids on a solid surface
- expresses the wettability of the solid
- measured through the denser phase, usually water





Complex interaction between water, air and solid phases

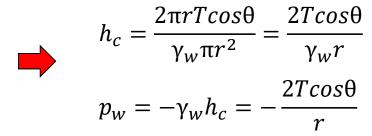
- The interface air-water near a solid surface has a curvature due to surface tension
- The water in the voids is similar to the water in a capillary tube.



T: Surface tension (force per unit of length) θ : contact angle (it depends on the characteristics of the liquid and the solid)

Vertical equilibrium of the water column

$$\gamma_w h_c \pi r^2 = 2\pi r T \cos \theta$$



The gauge water pressure is negative (p_w)

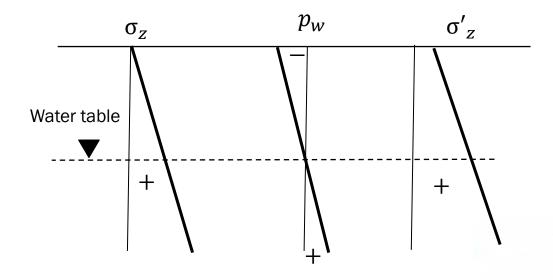
Young-Laplace equation

$$s = p_a - p_w = \frac{4Tcos\theta}{D}$$
 Suction s

$$D = 2r$$



A negative pore water pressure allows having non-zero effective stresses even at a zero depth.



The <u>sand castle</u> is an example of how the negative pore water pressure in nearly saturated states affect the mechanical properties of soils. The **Terzaghi's effective stress** can be considered still valid if the soil remains saturated even with negative pore water pressure

$$\sigma'_z = \sigma_z - p_w$$



The **shear strength** can be higher than zero even for a granular non-cemented soil at zero depth

$$\tau_f = c' + \sigma'_n tan\phi'$$



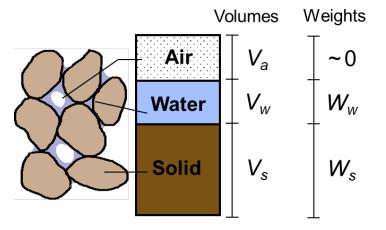
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3 stress variables

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Three-phase description

A complete description of the rheological behaviour of unsaturated soils requires

- Soil water retention behaviour
- Stress-strain behaviour

How Sr changes for a given suction?

How the soil deforms?

Stress-strain variables for constitutive modelling of unsaturated soils

Stress variables

Work-conjugate variables

$$\binom{\sigma'}{s}$$

$$\binom{\varepsilon}{Sr}$$



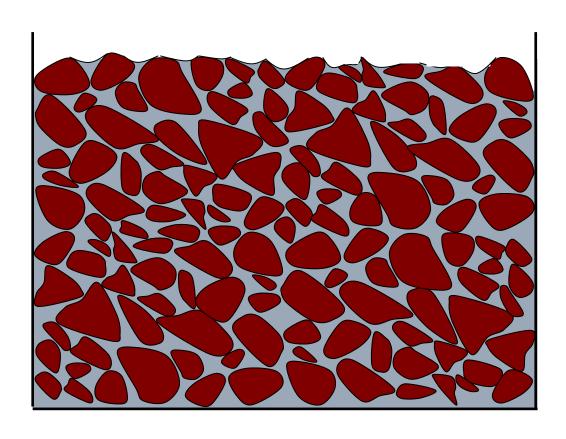
Hydraulic properties of geomaterials under unsaturated conditions

THE WATER RETENTION CURVE

THE PERMEABILITY



SATURATED STATE

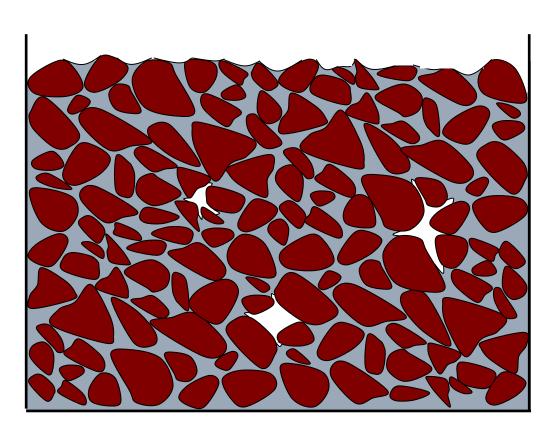


An initially saturated geomaterial is subjected to evaporation

- Menisci at the surface start to appear
- The pore water pressure decreases and the suction becomes higher than 0, s > 0
- The soil is still saturated, S_r = 1



QUASI - SATURATED STATE

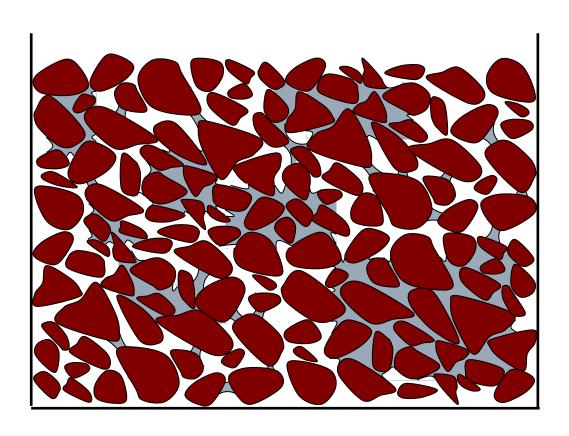


Indicative values of the degree of saturation 0.85-0.90 < Sr < 1

- The pore water pressure continues decreasing and the suction continue increasing, s>0
- More menisci appear, air cavities in the larger pores expand due to negative porewater pressure
- Gas phase is discontinuous, liquid phase is continuous
- The degree of saturation decreases, S_r<1</p>



PARTIALLY SATURATED STATE

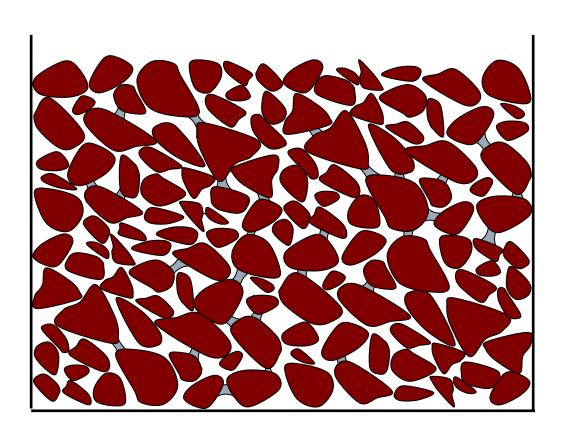


Indicative values of the degree of saturation $0.1 < S_r < 0.85-0.90$

- More menisci appear, smaller pores start to desaturate;
- At the interface, the maximum curvature of the menisci is reached, and the air enter
- The pore water pressure continues decreasing and the suction continues increasing, s>0
- Gas phase is continuous, liquid phase is continuous
- The degree of saturation decreases, S,<1



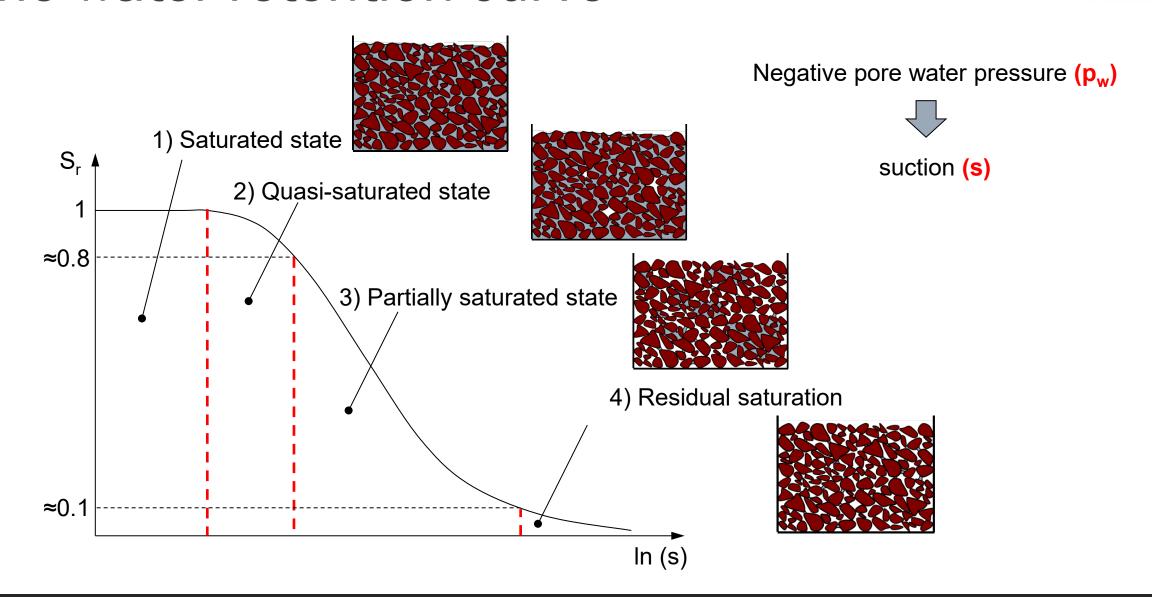
RESIDUAL STATE



Indicative values of the degree of saturation $S_r < 0.1$

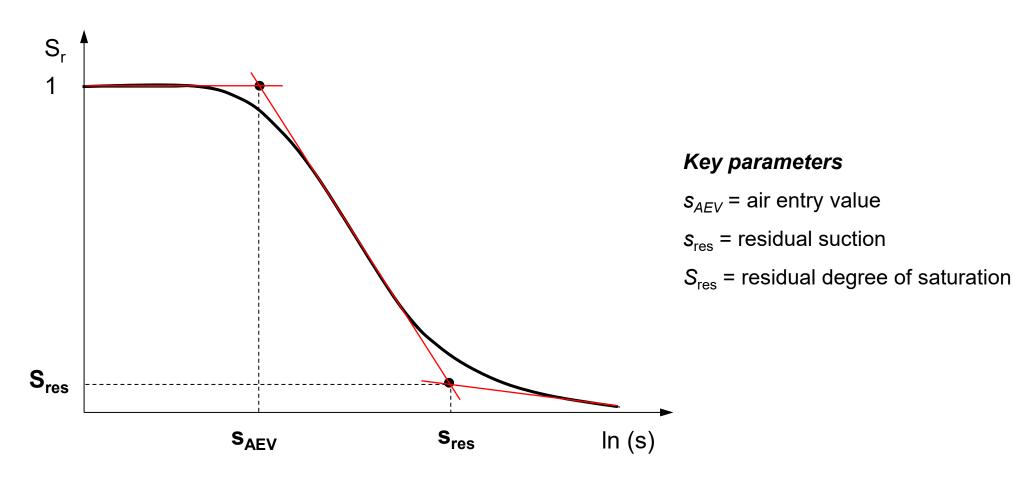
- There is water in some isolated pores
- The pore water pressure continues decreasing and the suction continues increasing, s>0
- Gas phase is continuous, liquid phase is discontinuous
- The degree of saturation does not decrease significantly







The relationship between the degree of saturation and the suction is called "water retention curve". It expresses the capacity of the soil to retain water





THE AIR ENTRY VALUE

- Value of suction from which the soil start to desaturate
- Function of the pores size: a lower suction is needed to desaturate a soil with larger pores

Young-Laplace equation

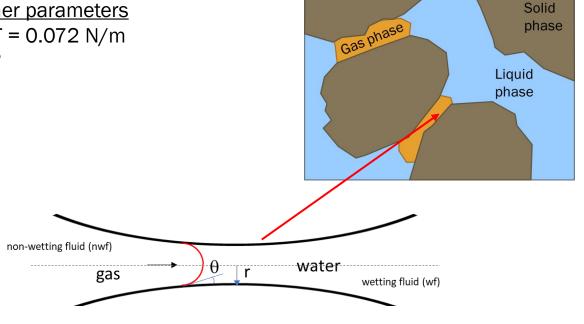
$$s = p_a - p_w = \frac{4T\cos\theta}{D}$$

D: pore diameter for different geomaterials

Values assumed for the other parameters Water-air surface tension, T = 0.072 N/m Water contact angle, θ = 0°

Order of magnitude of the air entry value depending on the characteristic pore size:

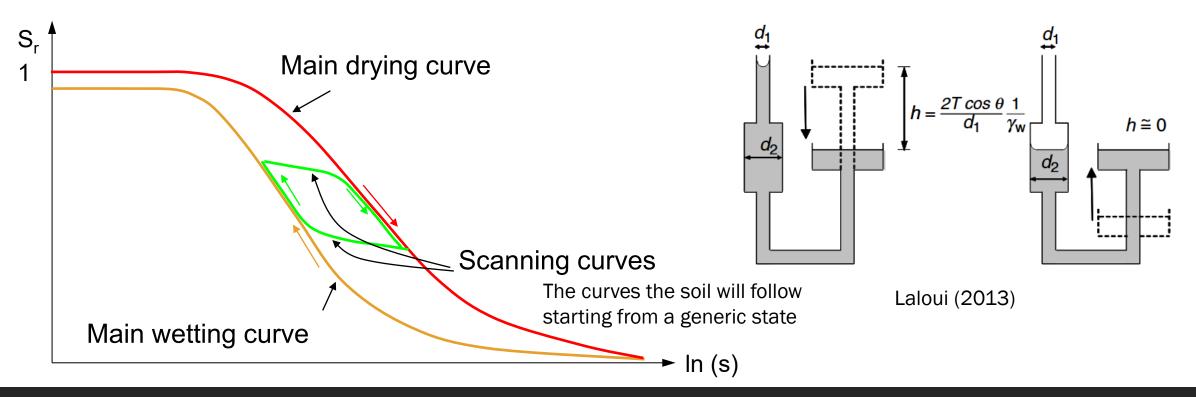
sand	silt	clay
(<i>d</i> _{max} =30 μm)	(d _{max} =3 μm)	(<i>d</i> _{max} =0.3 μm)
9.6 kPa	96 kPa	960 kPa





DRYING AND WETTING PROCESSES – HYSTERETIC BEHAVIOR

- Increase in suction is called "drying process"
- Decrease in suction is called "wetting process"
- The s-S_r relationship depends on the alternation of these processes



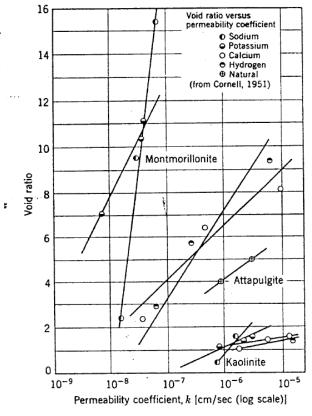
The permeability



The **coefficient of permeability** is the coefficient of proportionality between the flow rate and the hydraulic head gradient. **Darcy's law** (for the x-direction) gives:

$$v_{w,x} = -k_w \frac{\partial h_w}{\partial x}$$

For a given saturated soil, the coefficient of permeability mainly depends on the void ratio

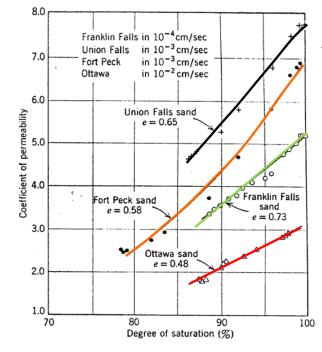


Lambe and Whitman, 1969

For an unsaturated soil, the Darcy's law is still valid but the coefficient of permeability depends on both the water content (or degree of saturation) and the void ratio.

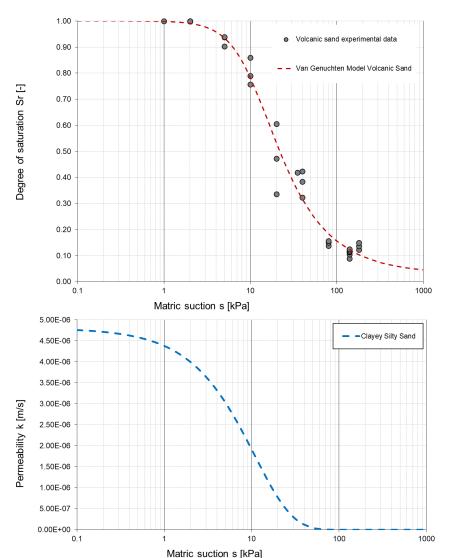
At higher degree of saturation, more connected pores are used by water to flow

For a given void ratio, the higher the degree of saturation, the higher is the permeability coefficient



WRC and permeability





Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + \left[\alpha (p_a - p_w) \right]^n} \right\}^m$$

 α ,n, m fitting parameters

The evolution of the coefficient of permeability with suction can be described for example by using the Gardner's model (1958).

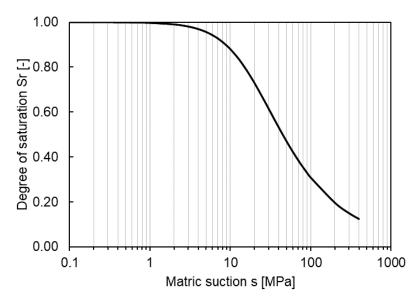
Gardner's model

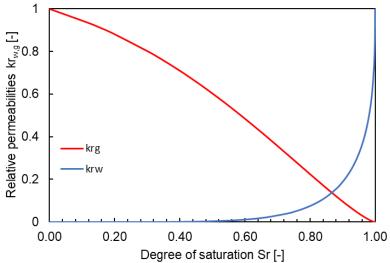
$$k = k_{sat}e^{-\alpha(p_a - p_w)}$$

α fitting parameter

WRC and permeability







Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

α ,n, m fitting parameters

Mualem, Y. 1976

$$k_{rw} = \sqrt{S_r} \cdot \left[1 - \left(1 - S_r^{\frac{1}{m}} \right)^m \right]^2$$
$$k_{rg} = \sqrt{1 - S_r} \cdot \left[1 - S_r^{\frac{1}{m}} \right]^{2m}$$

$$k_{rg} = \sqrt{1 - S_r} \cdot \left[1 - S_r^{\frac{1}{m}} \right]^{2m}$$

m fitting parameter

Generalized Darcy's law for partially saturated porous media Flux for a given fluid i, **q**_i

$$\boldsymbol{q_i} = -\frac{\boldsymbol{k_i}}{\mu_i} (\boldsymbol{\nabla} p_i + \rho_i \boldsymbol{g})$$

$$\mathbf{k}_i = \mathbf{K} k_{r,i}(S_r)$$

k: apparent permeability

μ: dynamic viscosity

p: pressure

 ρ : density

g: gravity

The intrinsic permeability K, is scaled by the relative permeability of the considered fluid i: $k_{r,i}$ fct(Sr_i)



Mechanical behaviour of unsaturated geomaterials

NET STRESS AND GENERALIZED EFFECTIVE STRESS

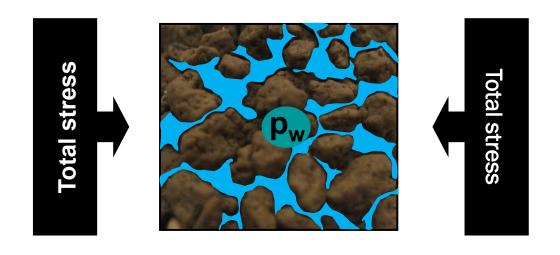
VOLUMETRIC BEHAVIOUR AND WETTING COLLAPSE

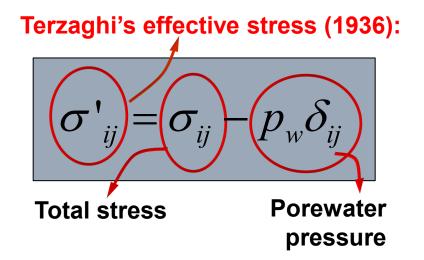
SHEAR STRENGTH

SWELLING SOILS & DESICCATION CRACKS



We have seen that in the case of **saturated specimens** with incompressible water and solid grains the effective stress is





SUCTION

The difference between the pressure of the air and the pressure of the water

$$s = p_a - p_w = \frac{2Tcos\theta}{r}$$

 $s\delta_{ij}$ suction stress tensor

NET STRESS

The difference between the external stress and the air pressure

$$\sigma_{net,ij} = \sigma_{ij} - p_a \delta_{ij}$$
 net stress tensor



If the porewater pressure is negative (suction > 0), two different situations must be distinguished:

Quasi-saturated state $s < s_{AEV}$ ($S_r = 1$)

Even if the pore water pressure is negative, the soil remains close to saturated

$$\sigma'_{ij} = \sigma_{ij} - p_{w}\delta_{ij}$$
 It is still valid

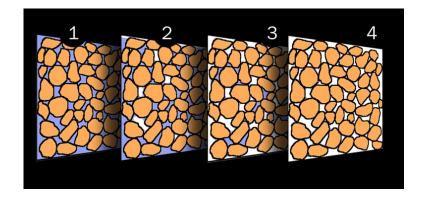
$$\rightarrow \sigma'_{ij} = \sigma_{ij} + |p_w| \delta_{ij}$$

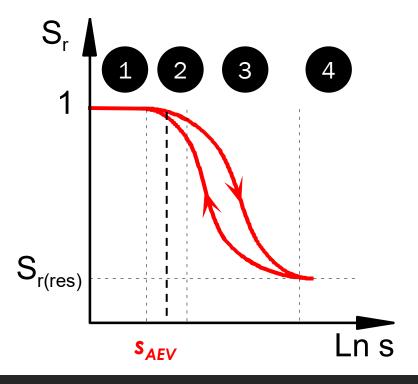
Partially saturated state $s > s_{AEV}(S_r < 1)$

The pore water pressure is negative and the soil is partially saturated

The effective stress concept needs to be generalized:

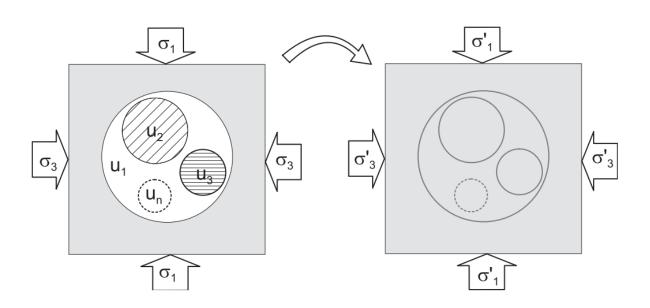
$$\rightarrow$$
 $\sigma'_{ij} = \sigma_{ij} - p_{equivalent} \delta_{ij}$







The effective stress with more than one fluid



Conversion of a multiphase and multi stress medium in an equivalent medium

For many geomaterials the following general expression of effective stress is valid

$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^{i} \alpha_{\beta} p_{\beta} \delta_{ij}$$

 σ_{ii} : total stress

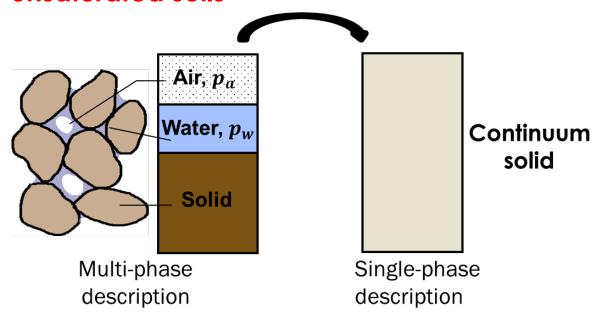
 β : generic fluid

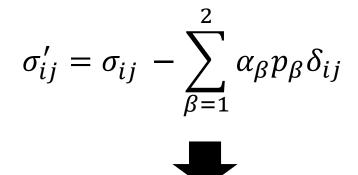
 α_{β} : scaling factor of the fluid β

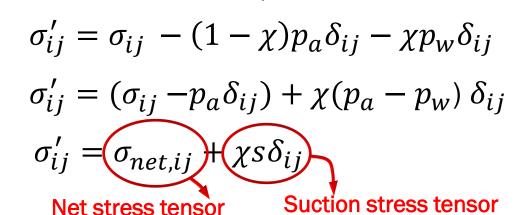
 p_{β} : pressure of the fluid β



The effective stress in the specific case of unsaturated soils







with
$$\chi = f(S_r)$$

if
$$\chi = Sr$$
 $\sigma'_{ij} = \sigma_{net,ij} + Sr \cdot s\delta_{ij}$

Stress-strain variables



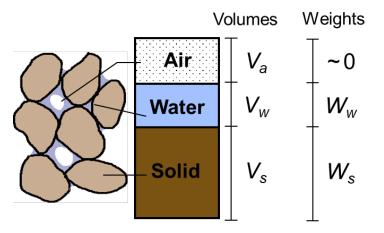
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Three-phase description

A complete description of the rheological behaviour of unsaturated soils requires

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How Sr changes for a given suction?

How the soil deforms?

Stress variables

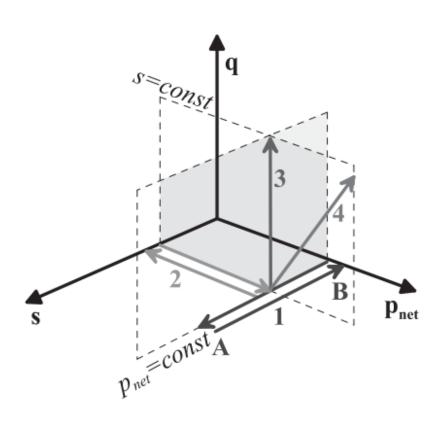
Work-conjugate variables

Stress-strain variables for constitutive modelling of unsaturated soils

$$\begin{pmatrix} \sigma' = \sigma_{net,ij} + Sr \cdot s\delta_{ij} \end{pmatrix} \qquad \begin{pmatrix} \varepsilon \\ Sr \end{pmatrix}$$

Possible stress paths



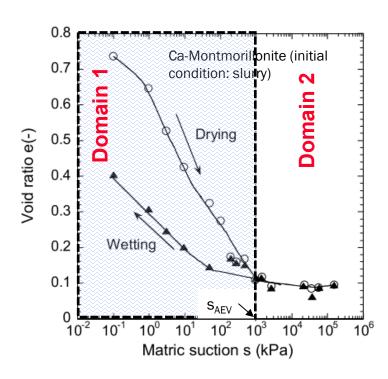


The possible stress paths to which a soil can be subjected can be highlighted in the 3D plane (s,p_{net},q)



<u>Isotropic hydraulic loading (the suction is increased at a constant net stress)</u>
Plane void ratio - suction

The **strains** induced by a drying or wetting process can be **elastic or elasto-plastic**. For some materials (usually if prepared from a slurry), we observe the following:

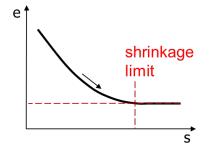


DOMAIN 1 - Elasto-plastic strains

If the suction is increased at a value lower than the air entry value and a wetting is performed, **NOT all the change in volume is recovered**

DOMAIN 2 - Elastic strains

If the suction is increased at a value higher than the air entry value and a wetting is performed, the **volume change** induced in the range of suction after the air entry value **is recovered**



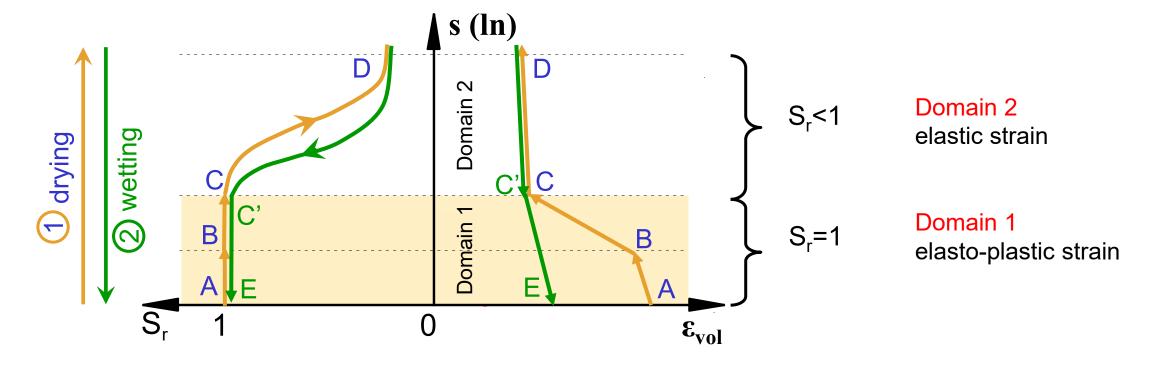
Shrinkage limit: value of suction starting from which no more significative variation in volume associated to an increase in suction are observed.



<u>Isotropic hydraulic loading (the suction is increased at a constant net stress)</u>

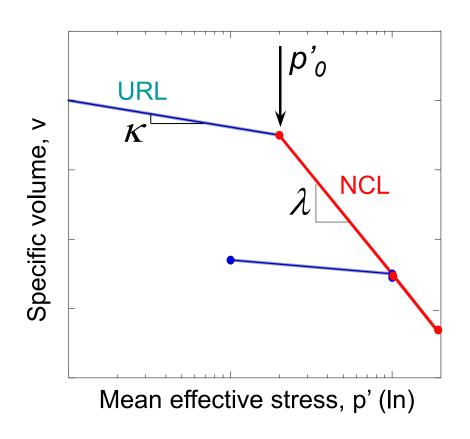
Plane suction-degree of saturation and suction-volumetric strain

The strains induced by a drying or wetting process can be elastic or elasto-plastic. For some materials (usually if prepared from a slurry), we observe the following:





<u>Isotropic mechanical loading – RECALL of the main equations and variables</u> Plane specific volume-mean effective stress



Normal Compression Line (NCL)

$$v = N - \lambda \ln p'$$

Unloading Reloading Line (URL)

$$v = v_{k} - k \ln p'$$

 λ : slope of the NCL (plastic compressibility) κ : slope of the URL (elastic compressibility) N: specific volume at p'=1 in the adopted measurement system ν_k : specific volume at p'=1 in the adopted measurement system

p'₀: Preconsolidation pressure



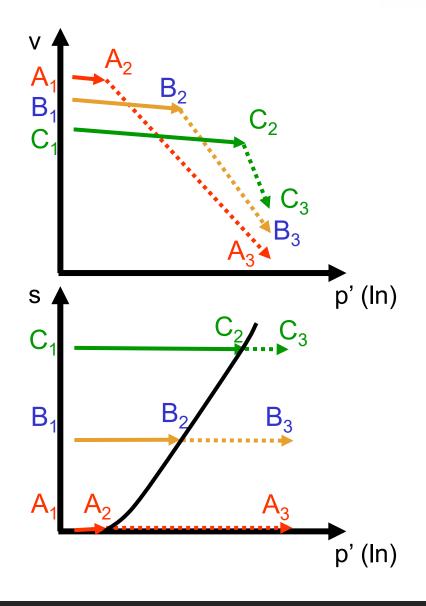
<u>Isotropic mechanical loading at constant suction</u>
Plane specific volume-mean effective stress

An increase of suction implies:

- an increase of the preconsolidation pressure (apparent hardening)
- a variation of the plastic compressibility
- elastic compressibility remains constant

Points A₂, B₂ and C₂ delimit the elastic domain for each path

The curve connecting all the preconsolidation pressure at the different suction values is the yield locus in the plane (s-p'). It is called **Loading-Collapse curve (LC)**.

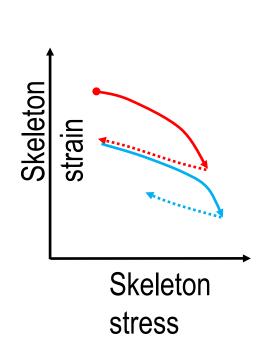


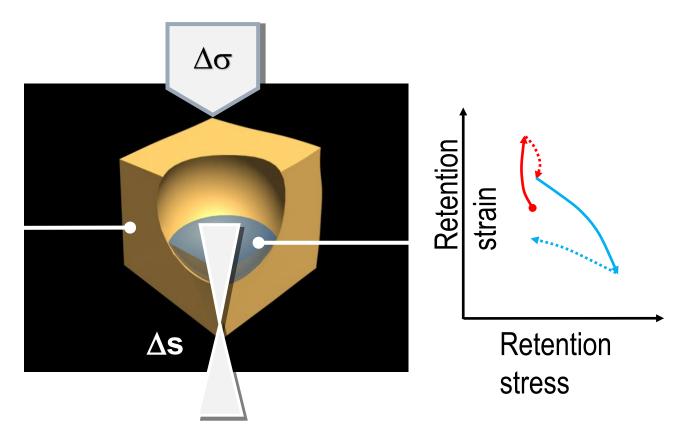


Skeleton straining and saturation changes are a response to

(a) External mechanical loading σ

(b) Variations in suction s





Irreversible processes in both cases

Wetting collapse

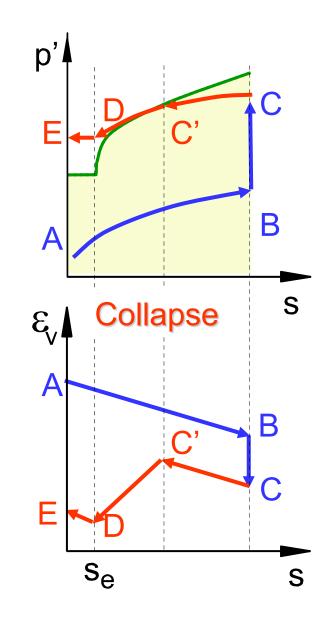
EPFL

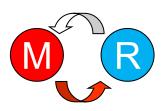
- AB: **drying** (p=const.)
- BC: mechanical consolidation (s=const.)
- CC': wetting elastic swelling
- C'D: wetting plastic collapse
- DE: wetting elastic swelling

— LC curve



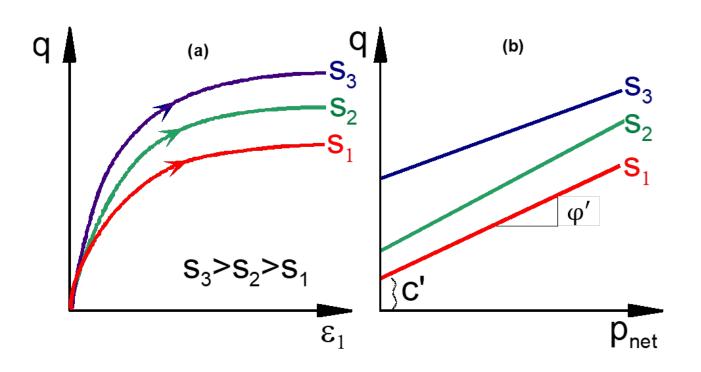
Elastic domain







Deviatoric mechanical loading at constant suction Planes q- ϵ_1 and q- ρ_{net}



Effects of suction on the shear strength

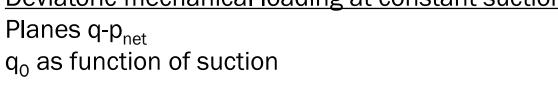
An increase of suction implies

- an increase of the cohesion intercept c'
- a variation of the shear strength angle φ'
- an increase of the overall shear strength



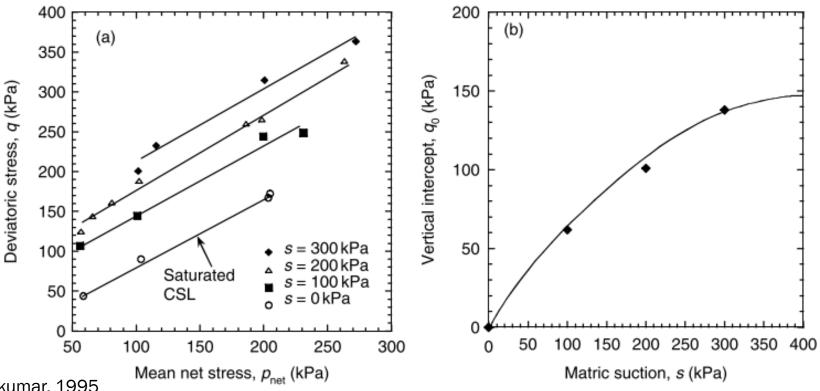
<u>Deviatoric mechanical loading at constant suction</u>

Planes q-p_{net}





- Apparent cohesion is suction dependent
- Friction angle remains fairly constant
- Increase of the overall shear strength



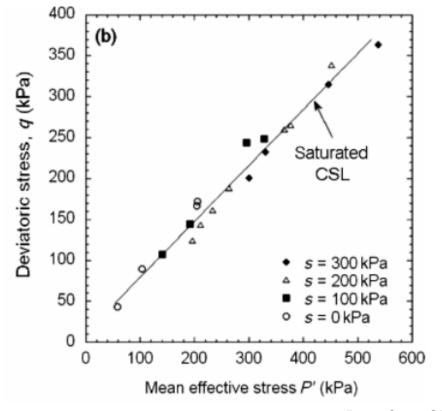
Data from Wheeler & Sivakumar, 1995



<u>Deviatoric mechanical loading at constant suction</u> Planes q-p'

$$p' = p_{net} + S_r s$$

$$p_{net} = p - p_a \qquad p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$



Data from Sivakumar, 1995

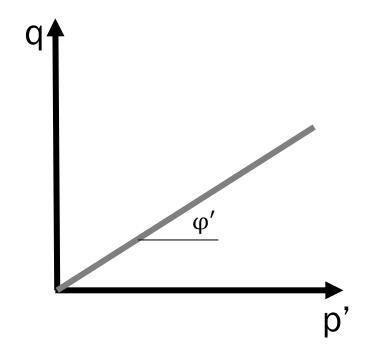
The adoption of a **suitable definition of effective stress** allows defining a **unique shear strength envelope** irrespective of the degree of saturation of the soil. **An increase of the mean effective stress,** related to an increase in **suction** and/or an increase of the **mechanical load,** results in an increase of the shear strength.



<u>Deviatoric mechanical loading at constant suction</u> Planes q-p'

$$p' = p_{net} + S_r s$$

$$p_{net} = p - p_a \qquad p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$



The adoption of a suitable definition of effective stress allows defining a unique shear strength envelope irrespective of the degree of saturation of the soil.

An increase of the mean effective stress, related to an increase in suction and/or an increase of the mechanical load, results in an increase of the shear strength

Conclusion



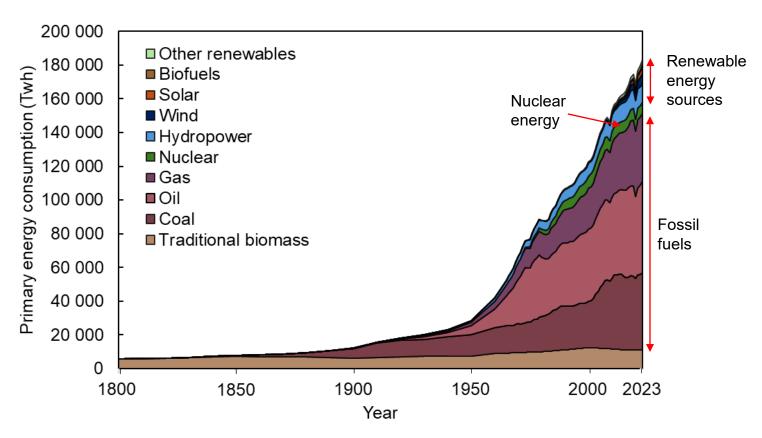
- Unsaturated soils are characterized by two fluids and a solid phase
- The **negative pore water pressure** is usually expressed in terms of **suction**
- The water retention curve describes the relationship between degree of saturation and suction
- The Darcy's law is still valid for unsaturated soils, the permeability is expressed in function of the degree of saturation
- The Terzaghi's effective stress is still valid if the pore water pressure is negative and the degree of saturation is close to 1. Otherwise, a more general expression has to be used
- The volumetric behaviour and the shear strength of a soil are affected by variation in suction



Unsaturated geomaterials in the context of nuclear waste disposal

Nuclear energy





Worldwide primary energy consumption in terawatt-hours (Twh) Data source: Energy Institute - Statistical Review of World Energy (2024); Smil (2017)

Energy has shaped human development

Nuclear energy accounts for 10% of worldwide electricity production (~32% in CH)

Significant challenges

- Risk of accidents (reduced by advances in technology)
- Radioactive waste management

Types of Nuclear Waste

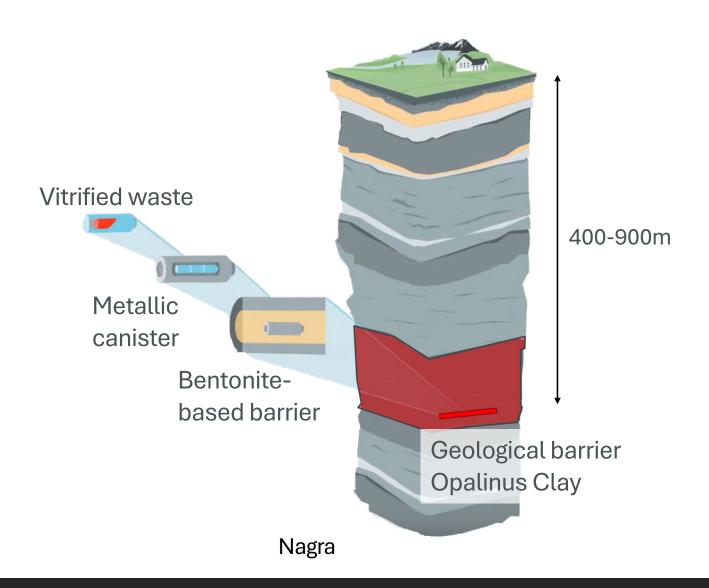


- 3% High-level Waste
- 7% Intermediate-level Waste
- 90% Low-level Waste

Source: World Nuclear Association

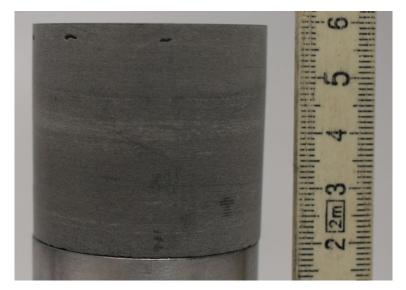
Swiss concept - Multi-barrier system





Opalinus Clay shale

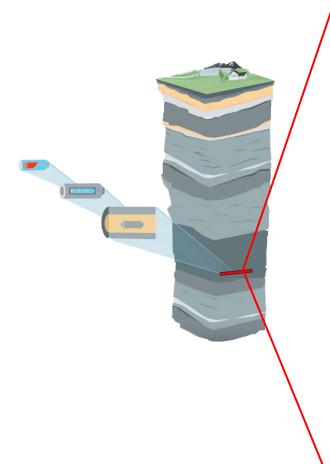
- Low permeability: 10⁻¹⁹-10⁻²¹ m²
- Self-sealing capacity
- High retention of radionuclides

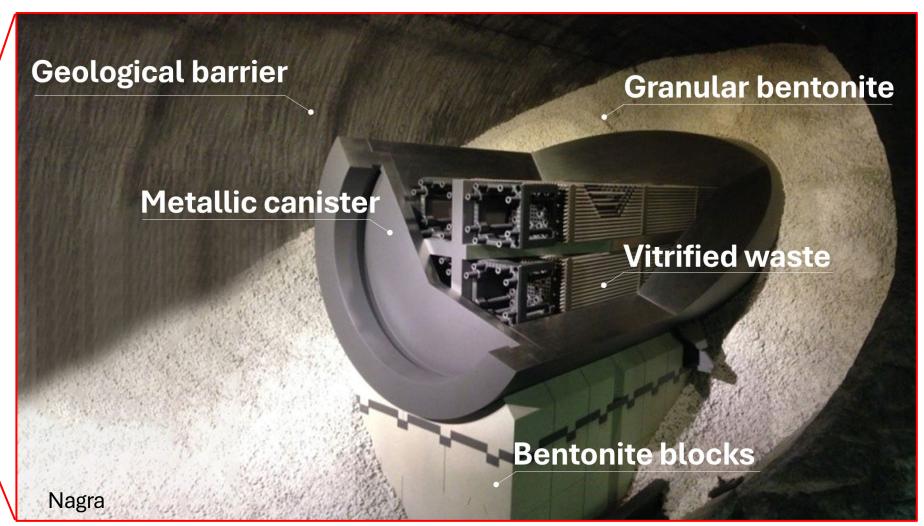


Picture of Opalinus Clay sample for triaxial test (EPFL/LMS)

Swiss concept - Multi-barrier system







Long-term multi-physical processes

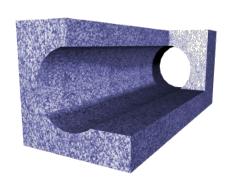


Excavation



Stress redistribution

~1 year



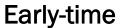
Ventilation



Consolidation

Desaturation

~10 years





Hydration from the far field

Heating from the canister

~100 years





Isothermal conditions restored

~1000 years Very-late-time



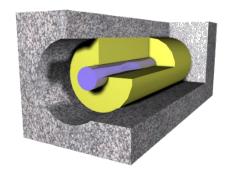


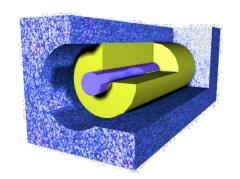
Gas generation and migration

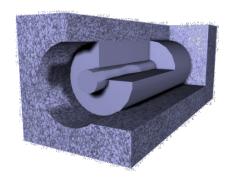
equilibrium

Towards a THM

~10,000 years

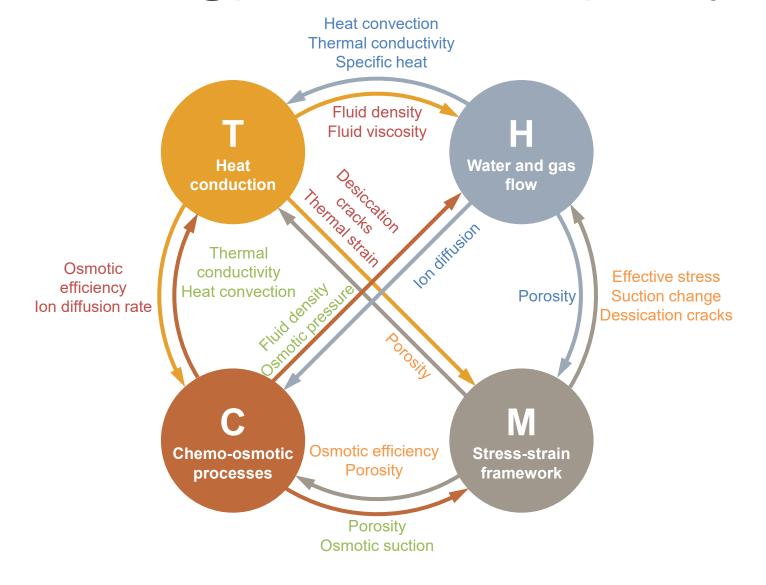






Overview of interacting phenomena in a repository





(T)HM framework





- Compositional approach
 - mass balance of species:
 - Water, air and solid



- Primary state variables:
 - Medium displacement: u
 - •Water pressure: p_w
 - •Gas pressure: p_a
 - ■Temperature: *T*

Balance equations



Mass balance of water (liquid water + vapour) - unknown: p_w

$$\frac{\partial}{\partial t}(\rho_w n S_r) + \operatorname{div}(\rho_w \mathbf{f}_l) - Q_w$$

$$+\frac{\partial}{\partial t}[\rho_v n(1-S_r)] + \operatorname{div}(\mathbf{i}_v + \rho_v \mathbf{f}_g) - Q_v = 0$$

Mass balance of air (dry air + dissolved air) – unknown: p_a

$$\frac{\partial}{\partial t} \left[\rho_a n (1 - S_r) \right] + \operatorname{div} \left(\rho_a \mathbf{f}_g + \mathbf{i}_a \right) - Q_a$$

$$+\frac{\partial}{\partial t}[\rho_a H_s n S_r] + \operatorname{div}(\rho_a H_s \mathbf{f}_l) - Q_{da} = 0$$

Mass balance of solids (Lagrangian approach):

$$\frac{\partial}{\partial t}\rho_S(1-n)V=0$$

Internal energy balance of the medium – unknown: T

$$\frac{\partial S_T}{\partial t} + L \frac{\partial}{\partial t} [\rho_v n (1 - S_r)] + \text{div}(\mathbf{f}_T)$$
$$+ L \cdot \text{div}(\mathbf{i}_v + \rho_v \mathbf{f}_g) - Q_T = 0$$

Momentum balance of the medium - unknown: u

$$\operatorname{div}(\mathbf{\sigma}) + \mathbf{b} = \mathbf{0}$$

Constitutive equations



Liquid water flow

Darcy's Law:

$$\mathbf{f}_l = -\frac{k_w k_{rl} \mathbf{I}}{\mu} \operatorname{grad}(p_w)$$
 $k_{rl} = Sr^n$

Kozeny-Carman Relation:

$$k_w = k_{w0} \frac{n^N}{(1-n)^M} \frac{(1-n_0)^M}{n_0^N}$$

Temperature dependent viscosity:

$$\mu_w = 0.6612(T - 229)^{-1.562}$$

Vapour diffusion:

$$\mathbf{i}_{v} = n(1 - S_r)\tau D\rho_g \operatorname{grad}\left(\frac{\rho_a}{\rho_g}\right) = -\mathbf{i}_a \quad D = 5.893 \times 10^{-6} \frac{T^{2.3}}{p_g}$$

Heat flux:

$$\mathbf{f}_{T} = -\Gamma \operatorname{grad}(T) + \left[c_{p,w} \rho_{w} \mathbf{f}_{w} + c_{p,a} (\mathbf{i}_{a} + \rho_{a} \mathbf{f}_{g}) + c_{p,v} (\mathbf{i}_{v} + \rho_{v} \mathbf{f}_{g}) \right] (T - T_{0})$$

Gas Pressure (ideal gas):

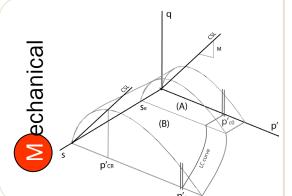
Dalton Law: $p_g = p_a + p_v$

$$\rho_a = \frac{p_a M_a}{RT} = \frac{(p_g - p_v) M_a}{RT} = \frac{p_g M_a}{RT} - \frac{\rho_v M_a}{M_v}$$

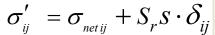
ACMEG-TS constitutive framework



Modelled behaviour



Stress variable



Generalised effective stress

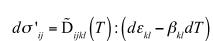


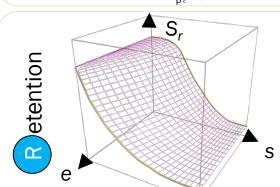


Skeleton strain









$$S = (p_a - p_w)$$

Matric suction

$$S_r$$

Degree of saturation

$$ds = A.dS_r$$



$$ds = A(T).dS_r$$

A: Retention properties

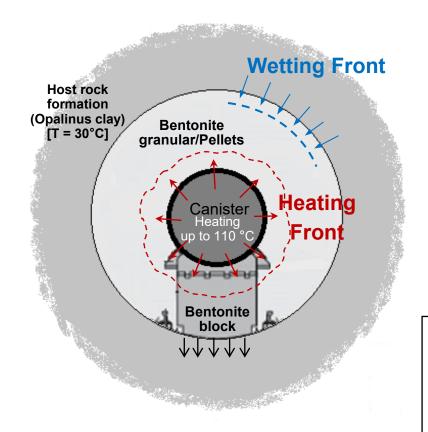






B. Francois and L. Laloui (2008)



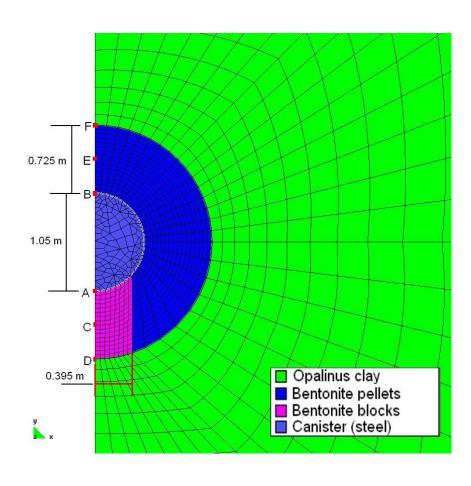


Heat emitting canister Heating front

Water saturation Wetting front



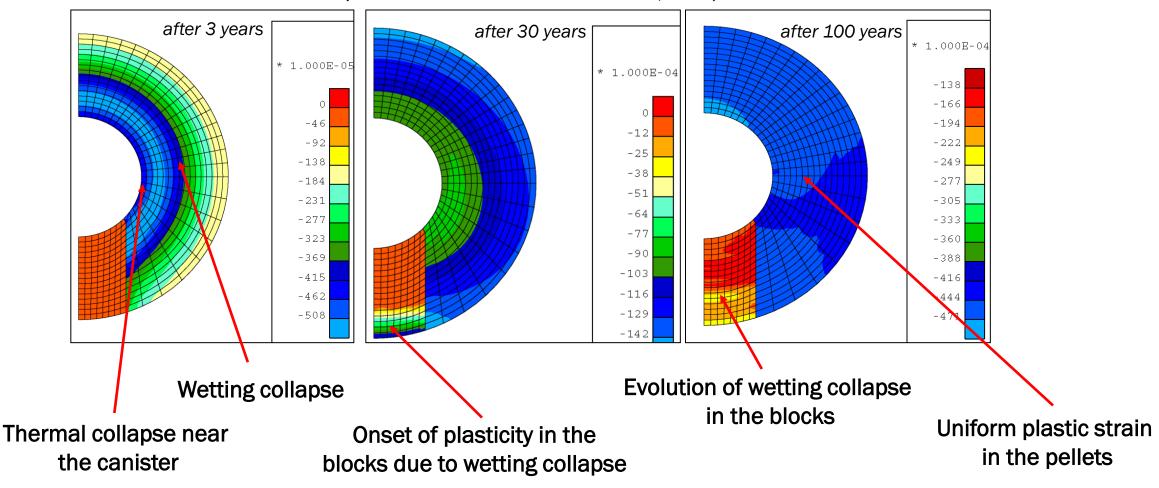
How heat and water transfer affect the mechanical behaviour of the buffer materials and the canister movement





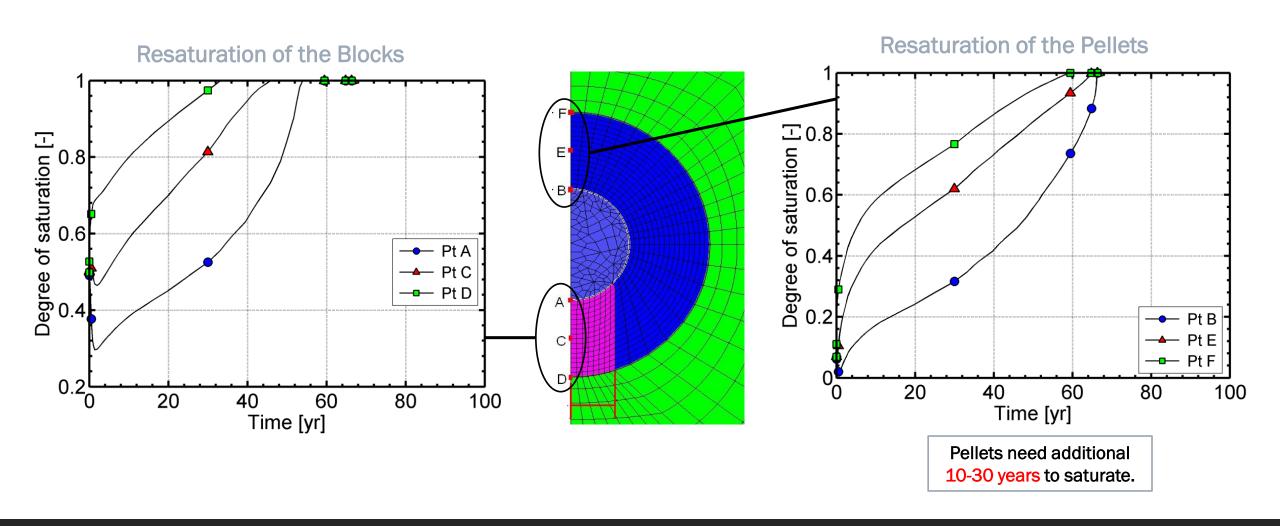
Plasticity due to heat and/or suction change

Maps of Plastic Volumetric Strains, in m/m



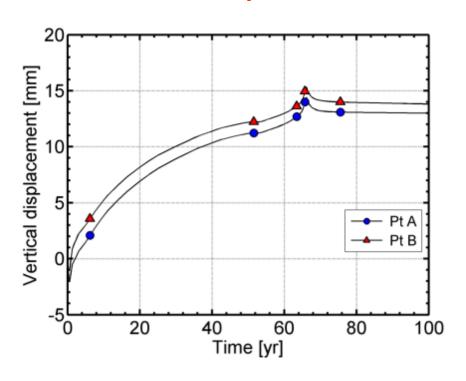


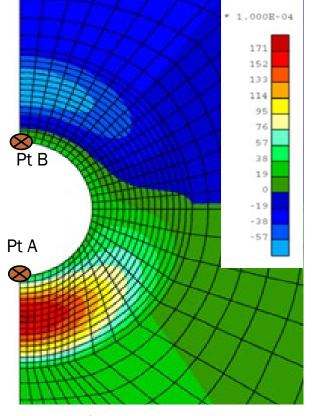
Resaturation of the bentonite buffer





Vertical displacement





Map of vertical displacement after 3 years, in m

Vertical movement of the canister

Bentonite swelling forces are stronger for blocks than for pellets

In this confined area, this competition creates an uplift of the canister



Thank you for your attention

